AMAT 327(Z): Elementary Abstract Algebra, Spring 2012 Midterm Exam, March 27

Name: ....

1] A] Please complete the following definition: we say that a function $f\colon X \to Y$ is *injective* if

B] Please complete the following definition: we say that a function $f\colon X \to Y$ is *surjective* if

C] Give two different examples of functions $f\colon \mathbb{R} \to \mathbb{R}$ which are injective but not surjective.

D] Give two different examples of functions $f\colon \mathbb{R} \to \mathbb{R}$ which are surjective but not injective.

E] Give two different examples of bijective functions $f\colon \mathbb{R} \to \mathbb{R}$.

2] Please complete the following definitions.

A] A *group* is a set G together with an operation $*$ satisfying the following axioms:

B] A group G is called *abelian* if

C] A non-empty subset H of a group G is a *subgroup* if and only if the following two conditions hold:

- for every $h, k \in H$,

- for every $h \in H$,

D] If G is a group and a is an element of G then the *cyclic subgroup generated by a* is the subgroup

$(a) =$

E] A group G is called *cyclic* if

3] Let $G = \mathbb{R} - \{0\}$ be the set of all non-zero real numbers. For all $a, b \in G$ define $a * b = -\dfrac{ab}{7}$.

Is it true or false that G with respect to this operation $*$ is a group?

State explicitly which group axioms hold and which ones (if any) fail.

4] In S_4 consider the subset $H = \left\{ \text{id}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \right\}$.

Is it true or false that H is a subgroup of S_4?

State explicitly which subgroup axioms hold and which ones (if any) fail.

5] Let G be a group and let a, b, c be elements of G such that $ab = ca$. Prove that for all $n \geq 1$, $ab^n = c^n a$.

6] Let G be a group and assume that for all $g \in G$ we have $g^{-1} = g$. Prove that G is abelian.

7] Let G be a group and $Z(G) = \{\, a \in G \mid \forall g \in G,\ ag = ga \,\}$ its center.

Prove that $Z(G)$ is a subgroup of G.

8] A] For an integer $n \geq 2$, what is the definition of U_n?

B] Find explicitly the number of elements in U_{495}.

C] Is [256] an element of U_{495}? Explain your answer, and if your answer is 'Yes' then compute the inverse of [256] in U_{495} and express it as $[a]$ with $0 < a < 495$.